

Bidding in sequential electricity markets: The Nordic case

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For electricity market participants trading in sequential markets with differences in price levels and risk exposure, coordinated bidding is highly relevant. We consider a Nordic power producer who engages in the day-ahead spot market and the near real-time balancing market. In both markets, clearing prices and dispatched volumes are unknown at the time of bidding. However, in the balancing market, the agent faces an additional risk of not being dispatched. Taking into account the sequential clearing of these markets and the gradual realization of market prices, we formulate the bidding problem as a multi-stage stochastic program. We investigate whether higher risk exposure can explain the hesitation, often observed in practice, to bid into the balancing market, even in cases of higher expected price levels. Furthermore, we quantify the gain from coordinated bidding, and by deriving bounds on this gain, assess the performance of alternative bidding strategies used in practice.

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1. Introduction

With 73% of the total physical power exchange in the Nordic region being traded at Nord Pool in 2011 (The Nordic blueprint (2011)), this is Europe's largest and most liquid market place for electricity. More specifically, Nord Pool operates the day-ahead spot market Elspot for the physical exchange of production and consumption. This market covers Norway, Sweden, Denmark, Finland and Estonia, and had 350 members and an impressive turnover of 294.4 TWh in 2011¹.

Although the spot market facilitates day-ahead balancing of expected production and consumption, real-time imbalances may still occur. It is the responsibility of the local electricity system operator, e.g. in Norway, Statnett, in Sweden, Svenska Kraftnät, and in Denmark, Energinet.dk, to ensure the physical balancing of supply and demand by activation of so-called regulating power. There exists a common Nordic market for regulating power, referred to as the balancing market, but trading is through the local transmission system operator. The balancing market has few members, e.g. in Denmark only 6, despite a total regulating power supply of approximately 0.75 TWh in 2011 (Personal communication with H. Parbo, Energinet.dk). The hesitation to enter this market is a major problem, reflected by a total demand for regulating power in Denmark of as much as 3 TWh². In general, the need for balancing services is expected to increase with the increasing growth in fluctuating renewable production, as pointed out by e.g. Holttinen et al. (2008, 2009).

The above electricity market design applies not only in the Nordic region, but analogies to day-ahead spot and near real-time balancing markets is also found e.g. in the Netherlands and Portugal/Spain, although with different bidding rules and market setups.

For electricity market participants able to engage in sequential markets such as the Nordic spot and balancing markets, coordinated bidding is highly relevant. Nevertheless, as already mentioned,

a hesitation to enter the balancing market can be observed in practice. This motivates the following research questions: Can the hesitation be explained by differences in price levels and risk exposure between the two markets? Is it profitable to hold back capacity in the spot market for the balancing market? If so, what is the gain from doing so?

To answer these questions, we consider a power producer who trades in a day-ahead spot market and an near real-time balancing market. In both markets, clearing prices and dispatched volumes are unknown at the time of bidding. However, in the regulating market, the agent faces an additional risk of not being dispatched. Taking into account the sequential clearing of these markets and the gradual realization of market prices, we formulate the bidding problem as a multi-stage stochastic program.

Our contribution is three-fold:

- We develop a three-stage stochastic programming model for coordinated bidding into two sequential markets, taking into account market price uncertainty and existing market rules. This model can be used for market exchange irrespective of production or consumption technology.
- When generating market price scenarios, we put special efforts into preserving autocorrelations and cross-correlations. Since the separate scenario sampling and reduction approaches suggested in the literature may alter correlations, we suggest an integrated approach.
- To assess the performance of alternative bidding strategies applied in practice, we derive bounds on the gain from coordinated bidding. These bounds can be computed without actually solving the three-stage stochastic coordination problem.

The paper is organized as follows. Section 2 first provides an overview of electricity market bidding in the literature. We proceed to introduce the Nordic electricity markets, including the spot and balancing markets in Section 3, and formulate a three-stage stochastic programming model for coordinated bidding into these markets in Section 4. Section 5 is concerned with the generation of market price scenarios that serve as input to the stochastic programming model. We derive bounds on the gain from coordinated bidding in Section 6, and numerically quantify this gain in Section 7. Finally, Section 8 concludes our analysis and discuss extensions of our model.

2. Electricity market bidding in the literature

The problem of optimal electricity market bidding is an optimization problem under uncertainty, given that the outcome of market clearing is unknown at the time of bidding. Naturally, the formulations of and solutions to the bidding problem found in the literature reflect the variety of approaches to optimization under uncertainty.

One strand of literature is based on optimal control and dynamic programming, and focus on the characterization and derivation of closed-form solutions to the bidding problem. An example is Anderson and Philpott (2002), who formulate a non-linear control problem and find necessary conditions for optimality. In their formulation, the authors make use of a so-called market distribution function, representing the probability that a generator is not fully dispatched at a given bid price and volume. For a price-taker, this is equivalent to the probability that the bid price exceeds the realized market price at a given volume, which is also what we use. Whereas an efficient approach to solving the non-linear problem remains an open question, Neame, Philpott and Pritchard (2003) derive optimality conditions for a hydroelectric reservoir with continuous output range, and solve the bidding problem by a discretization of the range and the application of dynamic programming. A similar approach is taken by Pritchard and Zakeri (2003), who likewise find offer curves for hydro reservoirs, and by Pritchard, Philpott and Neame (2005), who decompose the hydropower optimization into an inter-stage scheduling problem and an intra-stage bidding problem. In general, it is difficult to handle complex constraints and multiple state variables by

the optimal control approaches, and the bidding formulations may often account for operational restrictions only through the reward function, take into account only one market, and do not include the costs of non-compliance with the market commitments.

In contrast, mathematical programming formulations easily allow for various constraints, and furthermore, through the extension to stochastic programming, for multiple sources of uncertainty. Examples of bidding models for price-taking electricity producers are Contreras et al. (2002), Lu, Chow and Desrochers (2004), Ni, Luh and Rourke (2004), Fleten and Pettersen (2005), Ladurantaye, Gendreau and Potvin (2007) and Fleten and Kristoffersen (2007). These models include details such as ramping restrictions, capacity limits, storage balances, start-up costs, risk constraints etc. Reviews on optimal electricity scheduling and market exchange have been given by Wallace and Fleten (2003) and Kristoffersen and Fleten (2010).

Here, we extend the work of Fleten and Kristoffersen (2007) to sequential markets. This problem has already been addressed by Plazas, Conejo and Prieto (2005), who consider bidding into three sequential short-term markets. As in our case, the authors assume that producer is a price-taker in the day-ahead spot market and allow for price-making in the real-time balancing market. For the Nordic markets, contributions include Fosso et al. (1999), who consider production scheduling with a view towards the spot, regulating and futures markets, and Faria and Fleten (2011), who focus on the day-ahead Elspot market and intra-day Elbas market. To the best of our knowledge, very few have explicitly addressed the problem of coordinated bidding into the spot and balancing markets, the only example we could find being Olsson (2005). Whereas Fosso et al. (1999) model only price insensitive bids, Plazas, Conejo and Prieto (2005) do not distinguish between the process of bidding into the balancing market (before market clearing) and the settling of imbalances in this market (after market clearing), and Olsson (2005) assumes smooth bidding curves instead of the piece-wise linear curves dictated by the market rules. Furthermore, the above references neither account for the additional risk of not being dispatched in the balancing market, nor present an accurate modeling of the settlement mechanism, as we aim to.

With many details and the inclusion of uncertainty, mathematical programming models can be computationally hard and time consuming. Efforts to efficiently solve the bidding problem have been made by Klæboe (2011), who resort to Bender’s decomposition, and Nascimento and Powell (2009) and Loehndorf, Wozabal and Minner (2010), who apply approximate dynamic programming to integrate scheduling and bidding decisions for energy storage.

As an alternative to approximating the stochastic programming problem by cutting planes or simulation, we suggest to reduce computation time through careful generation of scenarios. Variations of scenario generation methods from the literature include Høyland and Wallace (2003) and Høyland and Wallace (2001), who propose moment/property matching by optimization or simulation. Another commonly used approach is to model the underlying stochastic processes, simulate a large number of sample paths/scenarios, and subsequently reduce this number by clustering. Central references on this method are Dupačová et al. (2003), Heitsch and Römisch (2003) and Heitsch and Römisch (2007) for two-stage programs, Heitsch and Römisch (2009) for multi-stage programs, and Gröwe-Kuska et al. (2003) for applications to power planning problems. For the particular case of bidding into sequential electricity markets, see Olsson and Söder (2008), who include the balancing market. Our scenario generation likewise relies on scenario sampling and reduction. However, whereas the existing literature handles sampling and reduction separately, we take an integrated approach designed to preserve the central properties of the stochastic processes.

From a practical point of view, there may be further challenges in implementing and solving the coordination problem (e.g. since this requires modeling software). This is finally our motivation for relating its solutions to alternative bidding strategies used in practice, and assessing the gain from coordination without actually solving it.

3. The Nordic short-term power markets

We consider two sequential markets common in many electricity market designs: A day-ahead and a near real-time market. In the Nordic region, these markets are the spot market Elspot and the balancing market.

The day-ahead market is the spot market for physical trading of production and consumption. As the name suggests, bidding takes place a day ahead of operation, when the market participants submit a set of price-volume bids for every hour of the following operation day (disregarding so-called block bids and flexible bids). For every such hour, the market interprets the set of bids as a points on a bidding curve. At closure around noon, the market is cleared, i.e. the demand and supply curves from all market participants are aggregated and the equilibrium is determined. Usually, bids are dispatched in merit order until aggregated demand and supply matches (however, taking into account transmission constraints). Bids can either be fully or partially (the marginal bid) accepted, and the marginal bid determines the market price (in the absence of bottlenecks). All trades are settled at this market price. Upon market clearing, the market prices for the following operation day are announced and the market participants are notified of their dispatch. The market participants are committed to comply with their dispatched volume, and hence must produce or consume accordingly.

Nevertheless, due to unforeseen events such as the inability to perfectly predict supply or demand or deliberate (strategic) noncompliance with the market commitments, expected and realized production and consumption may not fully match. Usually, an electricity system operator is responsible for ensuring physical balance of the power system. Hence, when imbalances occur, the system operator activates additional supply or demand by buying or selling so-called up and down regulating power. In situations of negative (resp. positive) imbalances, i.e. if real-time consumption (resp. production) exceeds production (resp. consumption), up (resp. down) regulation is activated.

Up and down regulation is traded in a near real-time market, here referred to as the balancing market. This market operates along many of the same principles as the spot market. Thus, for every hour of an operation day, the market participants submits a set of price-volume bids for up and down regulating power. However, bidding takes place during the operation day, with market closure being immediately prior to the delivery hour. This makes the market accessible only to agents that can quickly adjust production or consumption, also known as balance responsible parties. Furthermore, although bidding is on an hourly basis, the market clearing and subsequent dispatch occur continuously throughout the operation day, and is usually effectuated by the system operator irrespective of the origin of demand for regulation. The total volume dispatched, or equivalently the aggregated net demand for regulation, during the delivery hour determines the sign of the system imbalance, i.e. whether the system has generally been up and down regulated. This is used to establish the hourly balancing market price. In situations where the system is up (resp. down) regulated, it is the price of the most recently activated up (resp. down) regulation bid (as above, in the absence of bottlenecks). The balancing price is automatically higher (resp. lower) than the spot price, since less expensive bids have already been dispatched in the spot market. For this reason, the sign of the system imbalance is revealed from the spot and balancing market prices. If both up and down regulation bids are dispatched within the same delivery hour, special pricing principles apply.

For market participants who do not comply with their dispatched volumes in the spot and balancing markets (in case of trading in both markets), imbalances are penalized. In contrast to the process of balancing market bidding, imbalances are settled following the delivery hour, when realized production and consumption has been metered. Depending on the market design, a one-price or two-price balancing mechanism applies in the settlement. Under a one-price system, metered imbalances, whether positive or negative, are charged or paid the balancing market price. If a two-price system has been implemented, negative (resp. positive) imbalances are charged (resp.

paid) the balancing market price for imbalances of the same sign as net demand for regulating power, or equivalently the sign of the system imbalance, and otherwise the spot price. Clearly, this system has been designed to rule out incentives to deliberately create imbalances. As an example, in Western Denmark, the one-price system has been implemented for consumption, whereas the two-price system applies to production.

We consider a market participant who submits bidding curves to the spot and balancing markets over a time horizon of a day. The day is discretized into hourly time intervals according to the bidding practice in both markets. Although regulation is activated continuously throughout the delivery hour, we assume that only either up or down regulation is activated within the same hour, which makes it possible to settle all accounts on an hourly basis. The following assumptions holds for both markets. When the market participant can decide on both bid prices and volumes, the problem is non-linear, and we therefore discretize the price range, and fix a number of bid prices. Moreover, although we will later relax this assumption, we initially assume that the market participant take market prices as given. This makes it possible to determine a priori whether a bid is accepted or rejected. We assume that all bids are either fully accepted or rejected (i.e. the market participant never submits the marginal bid). Finally, we use system-wide market prices, and ignore any transmission constraints and potential bottlenecks.

As indicated above, both spot and balancing market prices are unknown at the time of bidding into the spot market. However, once the spot market has cleared, spot market prices are revealed. Likewise, balancing market prices are unknown at the time bidding into the balancing market, but are revealed once this market has cleared. Finally, both spot and balancing market prices are known when imbalances are settled. Taking into account the sequential clearing of the markets, we formulate the bidding problem a three-stage stochastic program. The first stage consists of spot market bidding, the second stage of balancing market bidding, and the third stage consists of imbalance settlement. Note that without operational uncertainty, the problem is a three-stage program. With such uncertainty, however, the problem in fact has 26 stages (the first stage consists only of spot market bidding, stage 2–25 of regulating market bidding, operation and settlement of imbalances, and the last stage consists only of imbalance settlement). However, as our main focus is spot market bidding, we use a three-stage approximation. By assuming a discrete distribution of market prices, the gradual realization of uncertainty can be represented by a so-called scenario tree.

4. The bidding problem

We use the following notation for the bidding problem. A 24 hour operation day is divided into hourly time intervals $[t-1, t], t = 1, \dots, T$ with $T = 24$, where the interval $[0, 1]$ represents the hour 00:00 – 01:00, $[1, 2]$ represents the hours 01:00 – 02:00 etc. In the following, however, we refer to the time interval $[t-1, t]$ simply as t .

We denote the random spot and balancing market prices in time interval t by ρ_t and μ_t , respectively. Accordingly, we assume that $\{\rho_t\}_{t=1}^T$ and $\{\mu_t\}_{t=1}^T$ are stochastic processes on some probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the sample space, \mathcal{F} is the σ -algebra, and \mathbb{P} is the probability measure. This probability space is equipped with a filtration given by the σ -fields $\mathcal{F}_1 = \{\emptyset, \Omega\} \subseteq \mathcal{F}_2 \subseteq \mathcal{F}_3 = \mathcal{F}$, where \mathcal{F}_2 is generated by $\{\rho_t\}_{t=1}^T$, and \mathcal{F}_3 is generated by $\{\rho_t\}_{t=1}^T$ and $\{\mu_t\}_{t=1}^T$. Hence, the σ -fields represents the information available in the first, second and third stages.

We represent the decisions of the sequential bidding problem by the multi-dimensional process $\{\mathbf{x}_t^{spot}, \mathbf{y}_t^{spot}, \mathbf{x}_t^{reg}, \mathbf{y}_t^{reg}, \mathbf{z}_t, \mathbf{q}_t\}_{t=1}^T$, where the processes (some of which are also multi-dimensional) $\{\mathbf{x}_t^{spot}\}_{t=1}^T$ and $\{\mathbf{x}_t^{reg}\}_{t=1}^T$ are the volumes bid in the spot and balancing markets, $\{\mathbf{y}_t^{spot}\}_{t=1}^T$ and $\{\mathbf{y}_t^{reg}\}_{t=1}^T$ are the corresponding volumes dispatched, $\{\mathbf{z}_t\}_{t=1}^T$ represents imbalances and $\{\mathbf{q}_t\}_{t=1}^T$ are the actual net production levels (a positive level indicates production, a negative level consumption). As for market prices, the sequence of decisions in the bidding problem forms a stochastic

process on $(\Omega, \mathcal{F}, \mathbb{P})$. We assume that this process is adapted to the filtration of σ -fields, or equivalently, that the decisions are non-anticipative. Non-anticipativity implies that decisions made in the first stage depend only on the information available in the first stage, and similarly for the second and third stages. Technically speaking, we assume that $\{\mathbf{x}_t^{spot}\}_{t=1}^T$ is \mathcal{F}_1 -measurable, $\{\mathbf{y}_t^{spot}, \mathbf{x}_t^{reg}\}_{t=1}^T$ is \mathcal{F}_2 -measurable, and $\{\mathbf{y}_t^{reg}, \mathbf{z}_t, \mathbf{q}_t\}_{t=1}^T$ is \mathcal{F}_3 -measurable.

For computational reasons, we assume that the processes $\{\rho_t\}_{t=1}^T$ and $\{\mu_t\}_{t=1}^T$ both follow a discrete distribution with a finite support given by a number of scenarios. With the above assumptions, the scenarios form a scenario tree. This scenario tree is based on a finite set of nodes \mathcal{N} . The root node $n = 1$ makes up the first stage, whereas the second and third stages consist of sets of nodes denoted by \mathcal{N}_2 and \mathcal{N}_3 , respectively. Apart from the root node, all nodes have an ascendant node and a set of descendant nodes. For node n , the immediate ascending node is denoted by n_- and the set of immediate descendants is denoted by $\mathcal{N}_+(n)$. Hence, for the three-stage scenario tree, we have that $n_- = 1$ for all $n \in \mathcal{N}_2$ and $n_- \in \mathcal{N}_2$ for all $n \in \mathcal{N}_3$, and moreover, $\mathcal{N}_+(1) = \mathcal{N}_2$ and $\mathcal{N}_+(n) \in \mathcal{N}_3$ for all $n \in \mathcal{N}_2$. Each node $n \in \mathcal{N}_2$ corresponds to a path of realizations $\{\rho_t^n\}_{t=1}^T$, referred to as a spot price scenario, and likewise each node $n \in \mathcal{N}_3$ corresponds to a spot price scenario and a balancing price scenario $\{\rho_t^n, \mu_t^n\}_{t=1}^T$. For all $n \in \mathcal{N}$ the corresponding probabilities are given recursively by $\pi^1 = 1$ and $\pi^n = \pi^{n/n_-} \pi^{n_-}$ for $n > 1$, where π^{n/n_-} is the probability that n is the descendant of n_- . The above assumptions further imply that the decisions of the bidding problem are assigned to nodes such that the first stage decisions \mathbf{x}_t^{spot} are made prior to any realizations, whereas the second-stage decisions $\{\mathbf{y}_t^{spot,n}, \mathbf{x}_t^{reg,n}\}_{t=1}^T, n \in \mathcal{N}_2$ are made following the observation of $\{\rho_t^n\}_{t=1}^T$ in the second stage, and the third-stage decisions $\{\mathbf{y}_t^{reg,n}, \mathbf{z}_t^n, \mathbf{q}_t^n\}_{t=1}^T, n \in \mathcal{N}_3$ following the realizations of $\{\rho_t^n, \mu_t^n\}_{t=1}^T$.

4.1. Spot market bidding

We consider both selling to and buying from the spot market such that the market participant submits a supply or demand curve for every hour of the following operation day. In any case, we assume that the bid prices are fixed, i.e. these are parameters, whereas the bid volumes are decision variables. We let the discretization of the price range be indexed by $1, \dots, I$. For a given hour t , the supply curve is then defined by the prices $p_{it}^+, i = 1, \dots, I$, where $p_{it}^+ \leq p_{i+1t}^+$ and $p_{1t}^+ = 0, p_{I+1t}^+ = +\infty$, and the volumes $x_{it}^{spot,+} \geq 0, i = 1, \dots, I$ (note that bid volumes are defined as the accumulated volumes at a particular bid price. Occasionally, we may however also refer to a bid as the incremental volume). When minimum and maximum bid volumes apply, we denote these by x^{min} and x^{max} , and include the decision variables $\delta_{it}^+ \in \{0, 1\}, i = 1, \dots, I$ such that $\delta_{it}^+ = 1$, when $x_{it}^{spot,+} > 0$, and $\delta_{it}^+ = 0$, when $x_{it}^{spot,+} = 0$. Likewise, for a given hour t , the demand curve is defined by the prices $p_{it}^-, i = 1, \dots, I$, where $p_{it}^- \leq p_{i-1t}^-$ and $p_{0t}^- = +\infty, p_{It}^- = 0$, and the volumes $x_{it}^{spot,-} \geq 0, i = 1, \dots, I$. The same minimum and maximum bidding volumes apply, with the corresponding decision variables $\delta_{it}^- \in \{0, 1\}, i = 1, \dots, I$. Obviously, if the producer could perfectly predict market prices, it would be unnecessary to submit bidding curves. However, market prices are unknown at the time of bidding. For this reason the bid volumes are node-independent. Once market prices are known, we can determine which (incremental) bids are accepted. We denote the total volumes dispatched in node n by $y_t^{spot,+,n}, y_t^{spot,-,n} \geq 0$, so that these decision variables represent the accumulated volume of accepted supply or demand bids, respectively.

Volumes bid are related to volumes dispatched through the bidding curve. Depending on the market rules, this curve may be a step-wise bidding curve, for which supply (resp. demand) bids are accepted if the bid price is below (resp. above) the market price. On the supply-side, this implies that

$$y_t^{spot,+,n} = x_{it}^{spot,+}, \text{ if } p_{it}^+ \leq \rho_t^n < p_{i+1t}^+, \quad i = 1, \dots, I, t = 1, \dots, T, n \in \mathcal{N}_2, \quad (1)$$

and on the demand-side

$$y_t^{spot,-,n} = x_{it}^{spot,-}, \text{ if } p_{it}^- \leq \rho_t^n < p_{i-1t}^-, \quad i = 1, \dots, I, t = 1, \dots, T, n \in \mathcal{N}_2. \quad (2)$$

Note that for a price-taker, we can determine a priori whether a bid is accepted or rejected, and express the relation between volumes bid and volumes dispatched as a linear constraint. Alternatively, the market rules may dictate a piece-wise linear bidding curve (this is the case in the Nordic market), which for supply is given by

$$y_t^{spot,+,n} = \frac{\rho_t^n - p_{it}^+}{p_{i+1t}^+ - p_{it}^+} x_{i+1t}^{spot,+} + \frac{p_{i+1t}^+ - \rho_t^n}{p_{i+1t}^+ - p_{it}^+} x_{it}^{spot,+}, \text{ if } p_{it}^+ \leq \rho_t^n < p_{i+1t}^+, \\ i = 1, \dots, I-1, t = 1, \dots, T, n \in \mathcal{N}_2,$$

and $y_t^{spot,+,n} = x_{1t}^{spot,+}$, $p_{1t}^+ \leq \rho_t^n$. For demand, it is

$$y_t^{spot,-,n} = \frac{\rho_t^n - p_{it-1}^-}{p_{it}^- - p_{i-1t}^-} x_{it}^{spot,-} + \frac{p_{it}^- - \rho_t^n}{p_{it}^- - p_{i-1t}^-} x_{i-1t}^{spot,-}, \text{ if } p_{it}^- \leq \rho_t^n < p_{i-1t}^-, \\ i = 2, \dots, I, t = 1, \dots, T, n \in \mathcal{N}_2,$$

and $y_t^{spot,-,n} = x_{1t}^{spot,-}$, $p_{1t}^- \leq \rho_t^n$. In any case, the market rules usually require bidding curves to be monotone (the supply curve is non-decreasing and the demand curve is non-increasing) such that

$$x_{it}^{spot,+} \leq x_{i+1t}^{spot,+}, \quad x_{it}^{spot,-} \leq x_{i+1t}^{spot,-}, \quad i = 1, \dots, I-1, t = 1, \dots, T. \quad (3)$$

Finally, when minimum and maximum bid volumes apply, we also have that

$$x_{it}^{min} \delta_{it}^+ \leq x_{i+1t}^{spot,+} - x_{it}^{spot,+} \leq x_{it}^{max}, \quad x_{it}^{min} \delta_{it}^{spot,-} \leq x_{i+1t}^- - x_{it}^- \leq x_{it}^{max}, \quad i = 1, \dots, I-1, t = 1, \dots, T.$$

For every hour of the operation day, the spot market profit is calculated as the market price times total volume dispatched. Thus, the expected total daily spot market profit is

$$\sum_{n \in \mathcal{N}_2} \sum_{t=1}^T \pi^n \rho_t^n (y_t^{spot,+,n} - y_t^{spot,-,n}).$$

Note that if the market participant is active only on supply-side, the bid volumes on the demand-side are fixed to zero, and vice versa.

4.2. Balancing market bidding

As for the spot market, we consider both selling to and buying from the balancing market, and hence the market participant submits a supply or demand curve for the following hour of operation. We likewise assume that bid prices are fixed a priori, and for ease of exposition, we use the same discretization of the price range. For a given hour t , the volumes bid are represented by the decision variables $x_{it}^{reg,+,n}, x_{it}^{reg,-,n} \geq 0, i = 1, \dots, I$, and the volumes dispatched in node n by $y_t^{reg,+,n}, y_t^{reg,-,n} \geq 0$. At the time of bidding into the balancing market, the spot market price is known, but the balancing market price is unknown. Once both the spot and balancing market prices are known, we can determine which bids are accepted.

We confine ourselves to step-wise bidding curves (the use of piece-wise bidding curves, however, follows the same arguments) and consider the relation between volumes bid and volumes dispatched. In the balancing market, up regulation bids are accepted if the bid price is below the market price *and* the system is up regulated, which is revealed from the spot and balancing market prices as

previously discussed. As mentioned above, in situations where the system is up regulated, the balancing market price is higher than spot price, and thus, we have that

$$y_t^{reg,+,n} = \begin{cases} x_{it}^{reg,+,n-}, & \text{if } p_{it}^+ \vee \rho_t^{n-} \leq \mu_t^n < p_{i+1t}^+ \\ 0, & \text{if } \mu_t^n < \rho_t^{n-} \end{cases} \quad i = 1, \dots, I, t = 1, \dots, T, n \in \mathcal{N}_3. \quad (4)$$

Again, for a price-taker, we can determine a priori whether a bid is accepted or rejected. Down regulation bids are likewise accepted if the bid price is above the market price *and* the system is down regulated, or equivalently, the balancing market price is lower than spot price. Therefore,

$$y_t^{reg,-,n} = \begin{cases} x_{it}^{reg,-,n-}, & \text{if } p_{it}^- \leq \mu_t^n < p_{i-1t}^- \wedge \rho_t^{n-} \\ 0, & \text{if } \mu_t^n > \rho_t^{n-} \end{cases} \quad i = 1, \dots, I, t = 1, \dots, T, n \in \mathcal{N}_3. \quad (5)$$

It should be remarked that in the balancing market, the agent faces an additional risk of not being dispatched, if offering up (resp. down) regulating in situations of a positive (resp. negative) system imbalance. As above, however, the bidding curves must be monotone, and so

$$x_{it}^{reg,+,n} \leq x_{i+1t}^{reg,+,n}, \quad x_{it}^{reg,-,n} \leq x_{i+1t}^{reg,-,n}, \quad i = 1, \dots, I-1, t = 1, \dots, T, n \in \mathcal{N}_2. \quad (6)$$

Finally, minimum and maximum bid volumes may also apply. Based on balancing market prices and dispatched volumes, the expected total daily regulating market profit can be calculated as

$$\sum_{n \in \mathcal{N}_3} \sum_{t=1}^T \pi^n \mu_t^n (y_t^{+,n} - y_t^{-,n}).$$

4.3. Settlement of imbalances

Imbalances occur when total spot and balancing market commitments do not comply with realized production and consumption. For a given hour t , positive and negative imbalances are represented by the decision variables $z_t^{+,n}, z_t^{-,n} \geq 0$, respectively, and net production by $q_t \geq 0$. Since imbalances are settled following the delivery hour, both spot and balancing market prices are known, and thus, the balancing costs are also known at the time of settlement.

The settlement of imbalances is formulated as

$$z_t^{-,n} - z_t^{+,n} = y_t^{spot,+,n-} - y_t^{spot,-,n-} + y_t^{reg,+,n} - y_t^{reg,-,n} - q_t^n, \quad t = 1, \dots, T, n \in \mathcal{N}_3. \quad (7)$$

Positive and negative imbalances may, respectively, be charged or paid the regulating market price. Under this one-price system, expected total daily balancing costs are

$$\sum_{n \in \mathcal{N}_3} \sum_{t=1}^T \pi^n \mu_t^n (z_t^{-,n} - z_t^{+,n}).$$

A two-price system implies that imbalances are charged or paid the balancing market price when these are of the same sign as system imbalances and otherwise the spot price. In this case, balancing costs sum to

$$\sum_{n \in \mathcal{N}_3} \sum_{t=1}^T \pi^n (\max\{\mu_t^n, \rho_t^{n-}\} z_t^{-,n} - \min\{\mu_t^n, \rho_t^{n-}\} z_t^{+,n}).$$

For ease of notation, we denote balancing prices by $\gamma_t^{1,-,n} = \gamma_t^{1,+,n} = \mu_t^n$ under a one-price system and by $\gamma_t^{2,-,n} = \max\{\mu_t^n, \rho_t^{n-}\}$, $\gamma_t^{2,+,n} = \min\{\mu_t^n, \rho_t^{n-}\}$ under a two-price system.

Finally, we account for costs of operation. Evidently, operations costs depend on the technology considered and any restrictions on production or consumption. For now, however, we assume that these costs are a function of net production

$$\sum_{n \in \mathcal{N}_3} \pi^n \mathcal{O}(q_1^n, \dots, q_T^n).$$

This function is the optimal value of some cost minimization subject to operational constraints, as will be clear from the following. We note that the objective function and constraints of the bidding problem are separable with respect to time periods. However, the cost minimization most likely introduces temporal dependencies through the operational constraints.

4.4. The stochastic programming problem

The three-stage stochastic programming problem of coordinated spot and balancing market bidding under balancing price mechanism i can be formulated as follows for $i = 1, 2$. The first stage consists of spot market bidding such as to maximize expected future revenues from the actual spot market dispatch and balancing market bidding, i.e.

$$z^i = \max \left\{ \mathbb{E} \left[Q_2(\mathbf{x}^{spot}) \middle| \mathcal{F}_2 \right] : (3) \right\}$$

In response to the realization of spot market prices, the second stage consists of balancing market bidding such as to maximize the expected difference between future revenues from actual balancing market dispatch and balancing costs, and so

$$Q_2(\mathbf{x}^{spot}) = \max \left\{ \rho(\mathbf{y}^{spot,+} - \mathbf{y}^{spot,-}) + \mathbb{E} \left[Q_3(\mathbf{x}^{reg}, \mathbf{y}^{spot}) \middle| \mathcal{F}_3 \right] : (1), (2), (6) \right\}.$$

Finally, upon the realization of balancing market prices, all imbalances are settled in the third stage, i.e.

$$Q_3(\mathbf{x}^{reg}, \mathbf{y}^{spot}) = \max \left\{ \mu(\mathbf{y}^{reg,+} - \mathbf{y}^{reg,-}) - (\gamma^{i,-} \mathbf{z}^- - \gamma^{i,+} \mathbf{z}^+) - \mathcal{O}(\mathbf{q}) : (4), (5), (7) \right\}.$$

Using the same notation as above, the so-called deterministic equivalent is

$$z^i = \max \left\{ \sum_{n \in \mathcal{N}_2} \pi^n \sum_{t=1}^T \rho_t^n (y_t^{spot,+,n} - y_t^{spot,-,n}) + \sum_{n \in \mathcal{N}_3} \pi^n \left(\sum_{t=1}^T (\mu_t^n (y_t^{reg,+,n} - y_t^{reg,-,n}) - (\gamma_t^{i,-,n} z_t^{-,n} - \gamma_t^{i,+,n} z_t^{+,n})) - \mathcal{O}(q_1^n, \dots, q_T^n) \right) \middle| (1) - (7) \right\},$$

for $i = 1, 2$. Disregarding the cost minimization, this problem is a linear or mixed-integer linear program.

4.5. Relaxing the price-taker assumption

So far, we assumed that the market participant is a price-taker. This is a valid assumption for many spot market participants, but may be questionable for the balancing market, where market participants tend to be larger players. We therefore relax the price-taker assumption by letting market prices respond linearly to the volumes dispatched. Denote by $\hat{\rho}_t^n$ and $\hat{\mu}_t^n$ the spot and balancing prices that realize if the producer or consumer does not participate in the market. The spot price decreases with increased supply or decreased demand volumes dispatched in the spot

market, whereas the balancing price respond to both spot and regulating market trades. Hence, we let

$$\begin{aligned}\rho_t^n &= \hat{\rho}_t^n - \alpha^{spot}(y_t^{spot,+,n} - y_t^{spot,-,n}), \quad t = 1, \dots, T, n \in \mathcal{N}_2, \\ \mu_t^n &= \hat{\mu}_t^n - \beta^{spot}(y_t^{spot,+,n} - y_t^{spot,-,n}) - \beta^{reg}(y_t^{reg,+,n} - y_t^{reg,-,n}), \quad t = 1, \dots, T, n \in \mathcal{N}_3,\end{aligned}$$

where $\alpha^{spot}, \beta^{spot}, \beta^{reg} > 0$ are parameters. As an approximation, we incorporate the price response only in the objective, and use $\hat{\rho}_t^n$ and $\hat{\mu}_t^n$ in the constraints and for the calculation of balancing costs. For $4\alpha^{spot}\beta^{reg} > (\beta^{spot})^2$, the resulting problem is a convex quadratic program (as above, disregarding the cost minimization).

Although we allow for price response in the following, for simplicity of notation, we denote the prices $\rho_t^n(y_t^{spot,+,n}, y_t^{spot,-,n})$ and $\mu_t^n(y_t^{spot,+,n}, y_t^{spot,-,n}, y_t^{reg,+,n}, y_t^{reg,-,n})$ simply by ρ_t^n and μ_t^n .

4.6. Operation

Since existing market rules (except for maybe the balancing price mechanism) applies to any market participant, the bidding model can be used irrespective of production or consumption technology. However, operations costs must be determined by cost minimization for the technology in question. The aim of the cost minimization problem is to produce or consume in accordance with the market commitments, while complying with operational constraints. For simplicity, we assume that this problem is deterministic, and hence, disregard operational uncertainty such as reservoir inflow, wind power production, availability of generating plants etc. We give a few examples.

4.6.1. Hydro-power In hydro-power operation, the problem is to determine water releases from a network of reservoirs such as to maximize the final value of water in storage subject to storage balancing and capacity restrictions. We denote by $j = 1, \dots, J$ the reservoirs, and assume for simplicity that these are serially connected. The decision variables $l_{jt}, v_{jt} \geq 0$ represent the storage and discharge levels for a given hour t . Upper and lower bounds on storage and discharge are denoted by the parameters $l_j^{min}, l_j^{max}, v_j^{min}, v_j^{max}$. Assuming constant water value and electricity generation efficiency, these are denoted by V_j and η_j , respectively. Finally, in addition to potential inflows from upstream reservoirs, we assume that all reservoirs have external inflows and denote the inflow in hour t by ν_{jt} . Then, the cost minimization problem is

$$\begin{aligned}\mathcal{O}(q_1^n, \dots, q_T^n) &= \max \left\{ - \sum_{j=1}^J V_j l_{jT} \mid \sum_{j=1}^J \eta_j v_{jt} = q_t^n, \quad l_{jt+1}^n = l_{jt}^n + \nu_{jt}^n + v_{j-1t}^n - v_{jt}^n, \right. \\ &\quad \left. l_j^{min} \leq l_{jt}^n \leq l_j^{max}, \quad v_j^{min} \leq v_{jt}^n \leq v_j^{max}, \quad j = 1, \dots, J, t = 1, \dots, T, n \in \mathcal{N}_3 \right\}.\end{aligned}$$

Note that the first set of constraints ensure compliance with the market commitments, and so these commitments can be viewed as a type of demand. For further details on the modeling of hydro-power operation, see for example Fleten and Kristoffersen (2008).

4.6.2. Thermal generation The thermal generation problem is concerned with the optimal operation of a number of production units. We denote these units by $j = 1, \dots, J$. Production levels for a given hour t are represented by the decision variables g_{jt} , and their upper and lower bounds by g_j^{min}, g_j^{max} . The variable operation cost is denoted by a_j . As a result, the cost minimization problem is

$$\begin{aligned}\mathcal{O}(q_1^n, \dots, q_T^n) &= \max \left\{ \sum_{t=1}^T \sum_{j=1}^J a_j g_{jt} \mid \sum_{j=1}^J g_{jt}^n = q_t^n, \right. \\ &\quad \left. g_j^{min} \leq g_{jt}^n \leq g_j^{max}, \quad j = 1, \dots, J, t = 1, \dots, T, n \in \mathcal{N}_3 \right\}\end{aligned}$$

As above, it should be remarked that the first set of constraints ensure compliance with the market commitments. This problem may be further subject to ramping restrictions, reserve constraints and include start-up costs.

By taking into account operational aspects by linear or mixed-integer linear modeling, the bidding problem remains a linear, mixed-integer linear or convex quadratic program.

5. Market price scenarios

We account for uncertainty in spot and balancing prices and for the sign of the system imbalance. As already indicated, however, the sign of the imbalance is revealed by spot and balancing prices, and so scenario generation reduces to describing a two-dimensional price process.

Spot and balancing prices exhibit strong autocorrelations and cross-correlations. The former are highly important for technologies with start-up costs, storage ability, ramping restrictions or other temporal dependencies in the operational constraints, whereas the latter are obviously relevant to market participants who engage in both the spot and balancing markets. We therefore put special efforts into preserving these correlation in the scenario generation.

Our scenario generation relies on scenario tree sampling and reduction. However, whereas the existing literature handles sampling and tree construction/reduction separately, we take an integrated approach. We describe spot and balancing prices by autoregressive processes that capture their autocorrelations. Moreover, since spot prices are known at the time of balancing market clearing, we include spot prices as exogenous variables in the balancing price process, and thereby also capture their cross-correlations. By sampling current prices conditional on previous ones, we preserve autocorrelations for both spot and balancing prices. Likewise, by sampling balancing market prices conditional on spot prices, we automatically induce the scenario tree structure and at the same time preserve cross-correlations. To accurately describe the distributions, we generate a large number of samples, and for computational reasons, we subsequently reduce this number. The idea is, however, to apply sampling and reduction in a stage-wise fashion in order not to alter cross-correlations in the process.

The stage-wise scenario generation approach can be summarized as follows:

1. Fit an autoregressive model to spot prices.
2. Fit an autoregressive model to balancing prices that includes exogenous spot prices.
3. Sample a fan of spot price scenario paths for the following 24 hours (second-stage scenarios), and reduce this fan by clustering.
4. For every spot price scenario path (second-stage scenario): Sample a fan of balancing price scenario paths for the following 24 hours (third-stage scenarios), and reduce this fan by clustering.

The problem of electricity spot price modeling is a subject of extensive study in the literature, see for example Nogales et al. (2002), Contreras et al. (2003), Garcia (2005) and Haldrup and Nielsen (2006), whereas the modeling of balancing market prices is relatively unexplored, a few references being Skytte (1999) and Olsson and Söder (2008). With only spot and balancing price data available, structural analysis (e.g. the inclusion of exogenous variables such as consumption), is infeasible. Like in most of the cited references, we instead fit autoregressive models, although ignoring more complex characteristics such as jumps, regime switching and volatility clustering. Prior to model fitting, we adjust for non-stationarity in prices by applying hourly differencing. Several other specifications were tested, including de-seasonalization and de-trending of the data, combinations of hourly, daily and weekly differencing, and logarithmic transformation of the data. However, none of these significantly improved stationarity.

We fit spot prices to the SARIMA(2,1,0) \times (1,0,1)²⁴ process

$$(1 - \phi_{24}^1 L)^{24} (1 - \phi_1^1 L - \phi_2^1 L) (1 - L) \rho_t = (1 - \theta_{24}^1 L)^{24} \epsilon_t^1,$$

where the L is the backshift operator, i.e. $L^k \rho_t = \rho_{t-k}$, $\phi_1^1, \phi_2^1, \phi_{24}^1$ and θ_{24}^1 are model parameters, and the innovations $\epsilon_t^1, t = 1, \dots, T$ are independent and identically distributed $N(0, \sigma_1^2)$. As above, several other model specifications were tested, but none were found superior to this model. The model describes hourly and daily cycles in the data (which are mainly due variations in consumption patterns).

The best fit to balancing prices is the ARMAX(1,0,0) process, including exogenous spot prices

$$(1 - \phi_1^2 L)(\rho_t - \mu_t) = \epsilon_t^2,$$

where ϕ_1^2 is a model parameter, and the innovations $\epsilon_t^2, t = 1, \dots, T$ are identically distributed $N(0, \sigma_2^2)$, mutually independent and independent to the above innovations.

To generate second-stage scenarios, we sample a number N of spot price paths $\{\rho_t^n\}_{t=1}^T, n = 1, \dots, N$ from the SARIMA process. We reduce the fan of scenario paths using a clustering algorithm in the spirit of Gröwe-Kuska et al. (2003). Hence, we partition the scenario paths into $k = |\mathcal{N}_2|$ clusters by iteratively assigning a path to the cluster in which the shortest distance to the remainder of the cluster is minimal. Each cluster is represented by the scenario of minimal distance to the remainder of the cluster. For every node of the resulting second-stage scenario tree $n \in \mathcal{N}_2$, we likewise generate third-stage scenarios by sampling a number N_n of regulating price paths $\{\mu_t^m\}_{t=1}^T, m = 1, \dots, N_n$ from the ARMAX process, and apply the clustering algorithm with $k = |\mathcal{N}_+(n)|$.

Upon scenario generation, we check for the existence of arbitrage opportunities. For a given time period t and $n \in \mathcal{N}_2$, we aim to avoid situations in which

$$\rho_t^n < \mu_t^m \text{ or } \rho_t^n > \mu_t^m, \text{ for all } m \in \mathcal{N}_+(n),$$

such that sure profits can be made by buying (resp. selling) in the spot market and down-regulating (resp. up-regulating) in the balancing market. Even for moderate size scenario trees, however, such situations turn out not to occur. We do allow for arbitrage opportunities that may exist when buying and selling in different time periods, and from which storage technologies also benefit in practice.

To fit the time series models in our Nordic case study, we use historical spot and balancing prices from January 1 2009 - December 31 2010 and obtained from the Danish transmission system operator Energinet.dk.

The generation of spot and balancing price scenarios is illustrated in Figures 1 and 2. For the two selected days December 14-15 2010, Figure 1 shows historical spot prices for the hours 1-24, conditional on which future price paths are sampled for hours 25-48. For the same two days, Figure 2 shows balancing prices, conditional on a given spot price path. The figures to the left display the sample paths prior to scenario reduction, and those to the right shows the result of the scenario reduction. For illustration purposes, the shown scenario fans consists of only 20 and 5 scenarios, before and after scenario reduction, respectively.

In our results for the Nordic case study, we generate 100 spot price scenarios and reduce these to 10. Conditional on each of these scenarios we likewise generate 100 balancing price scenarios and reduce these to 10. Figures 3 and 4 show the autocorrelations and cross-correlations prior to and as a result of the scenario reduction for December 15 2010. As can be seen from the figures, these correlations are very well preserved, in spite of having reduced the scenario tree from 100×100 leaves to only 10×10 . Further tests show that the means are likewise well very preserved, whereas the variances are reduced by the clustering as would be expected.

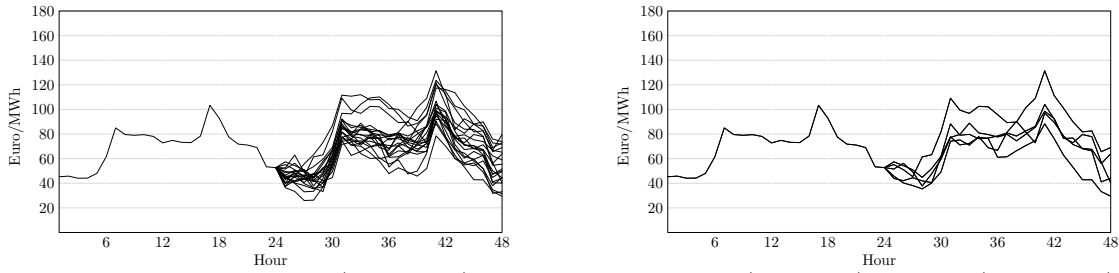


Figure 1 Historical spot prices (Hours 1-24) and sample paths before (to the left) and after (to the right) scenario reduction (Hours 25-48).

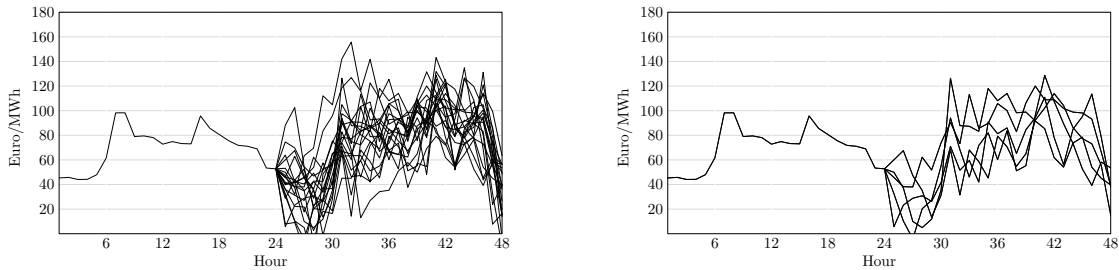


Figure 2 Historical balancing prices (Hours 1-24) and sample paths before (to the left) and after (to the right) scenario reduction (Hours 25-48).

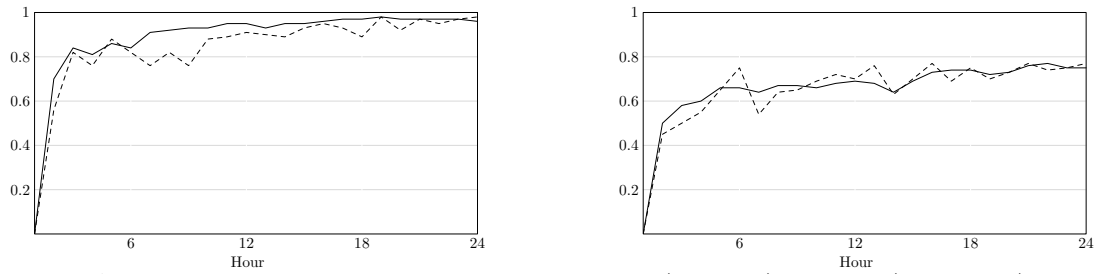


Figure 3 Autocorrelations in spot and balancing prices before (solid line) and after (dashed line) scenario reduction (Hours 1-24 corresponds to hours 25-48 in Figures 1 and 2).

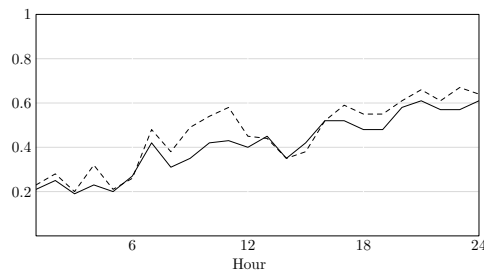


Figure 4 Cross-correlations between spot and balancing prices before (solid line) and after (dashed line) scenario reduction (Hours 1-24 corresponds to hours 25-48 in Figures 1 and 2).

6. The gain from coordinated bidding

In this section, we examine the profit gain from coordinated spot and balancing market bidding, or more specifically, we assess the performance of the separately derived bidding strategies often used in practice. We do this by deriving bounds on the profits obtained from coordinated and separate bidding.

Recall that the optimal value of the coordination problem under balancing price mechanism i is denoted by z^i for $i = 1, 2$.

To resemble that practitioners often derive the bidding strategies separately, we solve the spot and balancing market bidding problems in a sequential fashion. The spot market bidding problem under balancing price mechanism i is

$$\begin{aligned}
 z^{spot,i} = & \max \left\{ \sum_{n \in \mathcal{N}_2} \pi^n \sum_{t=1}^T \rho_t^n (y_t^{spot,+,n} - y_t^{spot,-,n}) - \sum_{n \in \mathcal{N}_3} \pi^n \left(\sum_{t=1}^T (\gamma_t^{i,-,n} z_t^{-,n} - \gamma_t^{i,+,n} z_t^{+,n}) \right. \right. \\
 & \left. \left. + \mathcal{O}(q_1^n, \dots, q_T^n) \right) \mid (1) - (3), \ z_t^{-,n} - z_t^{+,n} = y_t^{spot,+,n} - y_t^{spot,-,n} - q_t^n, \right. \\
 & \left. t = 1, \dots, T, n \in \mathcal{N}_2 \right\}, \tag{8}
 \end{aligned}$$

for $i = 1, 2$. This problem is a three-stage stochastic program. We denote the optimal volumes dispatched in the spot market by the vectors $\bar{\mathbf{y}}^{spot,+,n} = (\bar{y}_1^{spot,+,n}, \dots, \bar{y}_T^{spot,+,n})$ and $\bar{\mathbf{y}}^{spot,-,n} = (\bar{y}_1^{spot,-,n}, \dots, \bar{y}_T^{spot,-,n})$ for $n \in \mathcal{N}_2$.

In solving the problems sequentially, balancing bidding is conditional on the optimal volumes dispatched in the spot market. Hence, the balancing bidding problems under balancing price mechanism i are

$$\begin{aligned}
 z^{reg,i,n'}(\bar{\mathbf{y}}^{spot,+,n'}, \bar{\mathbf{y}}^{spot,-,n'}) = & \max \left\{ \sum_{n \in \mathcal{N}_+(n') \cap \mathcal{N}_3} \pi^{n/n'} \left(\sum_{t=1}^T (\mu_t^n (y_t^{reg,+,n} - y_t^{reg,-,n}) - (\gamma_t^{i,-,n} z_t^{-,n} - \gamma_t^{i,+,n} z_t^{+,n})) \right. \right. \\
 & \left. \left. - \mathcal{O}(q_1^n, \dots, q_T^n) \right) \mid (4) - (6), \ z_t^{-,n} - z_t^{+,n} = \bar{y}_t^{spot,+,n'} - \bar{y}_t^{spot,-,n'} \right. \\
 & \left. + y_t^{reg,+,n} - y_t^{reg,-,n} - q_t^n, \ t = 1, \dots, T, n \in \mathcal{N}_+(n') \cap \mathcal{N}_3 \right\},
 \end{aligned}$$

for $n' \in \mathcal{N}_2, i = 1, 2$. This problem is a two-stage stochastic program.

The expected profit from using the resulting spot market bids provides a lower bound on the profit from the coordination problem. However, to compute the expected profit from using the spot market bids, one must solve of $1 + |\mathcal{N}_2|$ bidding problems. A much less computationally expensive, though less tight, lower bound is provided by solving only the spot market bidding problem. This is formalized in the following.

PROPOSITION 1.

$$z^{spot,i} \leq \sum_{n \in \mathcal{N}_2} \pi^n \left(\sum_{t=1}^T \rho_t^n (\bar{y}_t^{spot,+,n} - \bar{y}_t^{spot,-,n}) + z^{reg,i,n}(\bar{\mathbf{y}}^{spot,+,n}, \bar{\mathbf{y}}^{spot,-,n}) \right) \leq z^i, \ i = 1, 2.$$

When the market participant is a price-taker, an upper bound on the profit in the coordination problem is provided by solving the spot market bidding problem, assuming a one-price balancing mechanism.

PROPOSITION 2. Assume that $\rho_t^n = \hat{\rho}_t^n, n \in \mathcal{N}_2$ and $\mu_t^n = \hat{\mu}_t^n, n \in \mathcal{N}_3$ (fixed prices). Then,

$$z^i \leq z^{spot,1}.$$

Note that the gain from coordinated bidding can be bounded without actually solving the coordination problem, but by solving at most two spot market bidding problems (under the one-price and two-price balancing mechanisms).

Propositions 1 and 2 apply irrespective of the balancing price mechanism. When the market participant is a price-taker, and under a one-price mechanism, however, we can show that one cannot obtain higher profits through balancing market bidding and, hence, it is sufficient to bid on the spot market.

COROLLARY 1. *Assume that $\rho_t^n = \hat{\rho}_t^n, n \in \mathcal{N}_2$ and $\mu_t^n = \hat{\mu}_t^n, n \in \mathcal{N}_3$ (fixed prices). Then, under a one-price balancing price mechanism,*

$$z^1 = z^{spot,1}.$$

In the following section, we investigate the quality of the bounds numerically.

7. Results

In this section, we quantify the gain from coordinated spot and balancing market bidding, and on the basis of the bounds derived in the previous section, assess the performance of alternative bidding strategies used in practice. Furthermore, we investigate whether higher risk exposure can explain the hesitation, often observed in practice, to bid into the balancing market, even in cases of higher expected price levels.

We illustrate the results from the bidding model in a Nordic case study. We consider the spot market at Nord Pool and the common Nordic balancing power market. As already mentioned, we use spot and balancing prices from January 1 2009 - December 31 2010 and obtained from the Danish transmission system operator, Energinet.dk. In the generation of market price scenarios, we work with 100 and 10 spot and balancing price scenarios, before and after scenario reduction, respectively. For the results of the scenario generation, see the previous section. We estimate the price response parameters from aggregate data and find their values to be $\alpha^{spot} = 0.0027$, $\beta^{spot} = 0.0057$ and $\beta^{reg} = 0.15$ Euro/MWh (using a currency rate of 7.5 DKK/Euro), which implies that the condition for convexity is satisfied. Since the aggregate price response is weak and have little effect on the results, we investigate the effect of a much stronger price response in our results and multiply the parameter values by 10. In the optimization of market exchange, the market rules dictate minimum bids of 0.1 MWh in the spot market, and minimum and maximum balancing market bids of 10 and 50 MWh, respectively, cf. the documents Trade at the Nordic spot market (2004) and The balancing market and balance settlement (2008). To illustrate the optimization of operation, the case study is based on a Norwegian hydro-power plant. We use real data when available, i.e. for reservoir storage and discharge capacities, generation efficiencies and external inflows. This plant has two serially connected reservoirs such that upstream water releases contribute to downstream inflows, and both reservoirs also have an external inflow. We assume that minimum and initial storage levels are 10% and 50% of full reservoir capacity, respectively. The water value function is assumed to be piece-wise linear and concave, with a water value of 45 Euro/MWh at 50% of full reservoir capacity.

The three-stage stochastic programming model has been implemented in the modeling software package GAMS, and run on a 1.7 GHz Intel Core i5 processor. Running times are less than a minute under the price-taker assumption, and so this model can easily be used for daily planning. It should be taken into account, however, that the modeling of operation could be much more complex, which will significantly increase running times and justify the scenario reduction. Moreover, the inclusion of price response increases running time to the proximity of 15 minutes.

The results from the bidding model can be found in Table 1. This table shows the results for a weekday in the middle of each month (the 15th, or the closest weekday) throughout 2010, and

assuming a price-taker. The results include empirical means and standard deviations of spot and balancing prices, profits under a one-price balancing system (the upper bound of Proposition 2, which we denote UB) and profits under a two-price balancing system, considering respectively spot market bidding and separate spot and balancing market bidding (the lower bounds of Proposition 1, denoted LB_1 and LB_2). It further includes the profit from coordinated bidding, the gain from coordinated bidding $((\text{Profit}-LB_2)/LB_2)$, the difference between the upper and lower bounds $((UB-LB_1)/LB_1)$, and finally the share of traded balancing market volumes out of total trading volumes.

We observe the following from the results. In our model, very large volumes are traded in the balancing market (in the reported results, the average is approximately 98%) in spite of lower average price levels than in the spot market (see columns 1,2, and 10 in Table 1). The explanation is the significant value of being able to defer the bidding decisions in the balancing market until an hour ahead of operation. We made further test runs in which the average balancing price is higher than the average spot price. For example, for December 2010, we increased balancing prices by 2.5% and found a balancing market volume share of 98.17%, by 5% and found a share of 99%, and by 10% and found a share of 100%. In spite of higher average price levels, not everything is necessarily traded in the balancing market. Obviously, this is due to the additional risk of not being dispatched in the balancing market. As an example, for December 2010, risk adjusted up regulation prices are only 43.92 Euro/MWh, whereas the spot and balancing market prices are 49.52 and 48.48 Euro/MWh, respectively. Hence, the risk of not being dispatched can to some extent explain a hesitation to hold back capacity for the balancing market in reality. It is worth noting, however, that the cost of this additional risk is relatively small compared to the value of waiting, and so for price increases of 10%, everything is indeed traded in the balancing market.

The profitability in our bidding model of holding hold back capacity for the balancing market is also confirmed from the bidding curves. In Figure 5, we show examples of optimal spot market supply curves for two selected hours, using separate and coordinated bidding strategies, respectively (Note that the small number of steps on the bidding curves is due to the simplistic modeling of hydropower operation, assuming a constant efficiency and a small number of line segments to define the piece-wise linear water value function). It is clear from the figure that very large volumes are traded in the balancing market.

Now, although coordinated bidding is profitable, we observe that the gain from this is relatively small (in the reported results, the average is around 2%, see columns 7 and 8 in Table 1). Our results suggest that it is rather this small gain than the additional risk that explains the hesitation to enter the balancing market. In reality, however, the hesitation may also be caused by substantial transaction costs (e.g. learning costs) for new entrants in the balancing market. Furthermore, to prohibit speculation in the balancing market, existing market rules require that physical exchange of power *mainly* takes place in the spot market, although no explicit restrictions apply to the volume allowance in the balancing market. Our model does not capture these aspects. To further investigate the size of the gain from coordinated bidding, we made additional test runs in which we increase the difference between spot and balancing price levels. We found that the larger this difference, the larger the gain. For example, for December 2010, we increased the difference by 125% and found a gain of 1.42%, by 150% and found a gain of 1.87%, and by 200% and found a gain of 2.92%. The results indicate that coordinated bidding may become increasingly important in a future power system, given that the expected growth in fluctuating renewable production is expected to increase the value of balancing power.

Finally, we compute the bounds on the gain from coordinated bidding derived in Propositions 1 and 2 (see columns 3-6 in Table 1). Upper and lower bounds are provided by the profits of the spot market bidding problem under the one-price and two-price systems, respectively, and the other lower bound by the profit from separate spot and balancing market bidding. In most cases, we find the difference between the upper and lower bounds to be a reasonable indicator for the gain

Table 1 Results for 2010, assuming a price-taker. The table shows empirical means and standard deviations of spot and balancing prices; profits under a one-price balancing system (UB) and profits under a two-price balancing system, considering respectively spot market bidding (LB₁), separate spot and balancing market bidding (LB₂) and coordinated bidding; the gain from coordinated bidding; the difference between the upper and lower bounds; and finally traded balancing market volumes out of total trading volumes.

	One-price		Two-price		Profit, sep.	Profit, cor.	Gain	Gain, pct.	(UB-LB ₁)/LB ₁	Bal. vol.
	Spot price	Bal. price	Profit	Profit, spot						
	Euro/MWh	Euro/MWh	Euro	Euro						
Jan	49.52 (17.40)	48.48 (27.55)	8292	7757	7798	7967	169	2.17	6.89	61.41
Feb	39.66 (16.58)	40.59 (26.34)	7448	6887	6972	7207	235	3.37	8.14	68.94
Mar	30.84 (18.09)	30.51 (27.19)	7001	6456	6549	6710	160	2.44	8.44	61.81
Apr	51.99 (16.58)	50.86 (26.04)	8504	8043	8050	8187	137	1.71	5.73	47.96
May	42.98 (19.77)	42.56 (27.99)	7429	6878	6934	7189	255	3.68	8.02	71.19
Jun	60.15 (19.48)	59.97 (28.65)	8941	8502	8507	8645	139	1.63	5.16	51.61
Jul	37.81 (19.40)	38.63 (27.53)	7764	7296	7344	7563	219	2.98	6.42	68.40
Aug	33.81 (17.06)	32.26 (25.79)	7502	6735	6786	6921	135	1.98	11.39	50.42
Sep	47.42 (13.36)	47.45 (24.73)	8168	7712	7736	7901	164	2.12	5.84	56.00
Oct	52.93 (17.59)	51.17 (26.62)	8864	8303	8308	8438	130	1.57	6.75	45.78
Nov	57.53 (12.80)	56.41 (23.97)	9216	8767	8768	8864	97	1.10	5.11	51.40
Dec	74.66 (19.96)	73.88 (28.97)	10469	10055	10056	10161	104	1.04	4.11	36.88

Table 2 Results for 2010, allowing for price response. The terminology is the same as in Table 2.

	One-price		Two-price		Profit, sep.	Profit, cor.	Gain	Gain, pct.	Bal. vol.
	Spot price Euro/MWh	Bal. price Euro/MWh	Profit Euro	Profit, spot Euro					
Jan	49.52 (17.40)	48.48 (27.55)	8240	7748	7779	7855	76	0.97	30.00
Feb	39.66 (16.58)	40.59 (26.34)	7410	6879	6942	7061	119	1.72	42.48
Mar	30.84 (18.09)	30.51 (27.19)	6944	6449	6514	6598	84	1.29	34.78
Apr	51.99 (16.58)	50.86 (26.04)	8446	8034	8039	8099	60	0.75	20.37
May	42.98 (19.77)	42.56 (27.99)	7392	6870	6907	7042	135	1.95	42.57
Jun	60.15 (19.48)	59.97 (28.65)	8891	8493	8497	8555	58	0.68	20.80
Jul	37.81 (19.40)	38.63 (27.53)	7733	7287	7325	7425	100	1.36	38.79
Aug	33.81 (17.06)	32.26 (25.79)	7425	6727	6762	6829	67	0.98	25.25
Sep	47.42 (13.36)	47.45 (24.73)	8113	7703	7722	7798	76	0.98	25.79
Oct	52.93 (17.59)	51.17 (26.62)	8804	8294	8297	8350	53	0.64	20.56
Nov	57.53 (12.80)	56.41 (23.97)	9163	8758	8759	8791	33	0.37	16.63
Dec	74.66 (19.96)	73.88 (28.97)	10408	10046	10047	10086	40	0.40	17.94

from coordination. In particular, assuming a practitioner already solves the spot market bidding problem, he/she may assess the profitability of entering the regulating market and the gain from coordinated bidding without actually solving the coordination problem, but merely by solving two variations of this problem (under the one-price and two-price balancing mechanisms). If the difference between upper and lower bounds is found to be small, there is no need to formulate and solve the coordination problem.

For a price-taker under a one-price balancing mechanism, the upper and lower bounds collapse, and the value of entering the balancing market is zero, cf. Corollary 1. Thus, this market design provides no incentives for agents to relieve system imbalances. However, the price-taker assumption is questionable in reality, and so we allow for price response in the results of Table 2. We observe that much smaller (although still substantial) volumes are traded in the regulating market, when prices decrease (resp. increase) with up-regulation (resp. down-regulation). For this reason, the gain from coordination is smaller for a price-making market participant than for a price-taker.

8. Conclusion and discussion

In this paper, we proposed a three-stage stochastic programming model for coordinated bidding into two sequential markets, namely the Nordic spot and balancing markets, and made special efforts to generate market price scenarios that preserve both autocorrelations and cross-correlations. Our main objective was to quantify the gain from coordinated bidding, and we derived bounds on this gain that can be computed without actually solving the coordination problem.

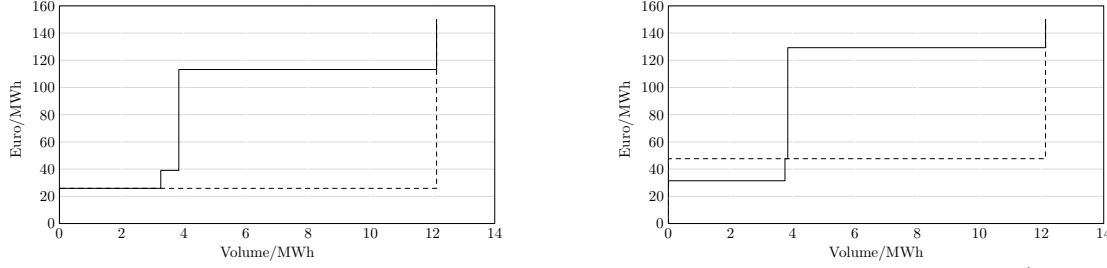


Figure 5 Spot market bidding curves for two selected hours of an operation day, using separate (dashed line) and coordinated (solid line) bidding.

For our Nordic case study, the bidding model indicates that it would be profitable to hold back capacity in the spot market for the balancing market. On one hand, it is optimal to trade very large volumes in the balancing market due to the value of being able to defer bidding decisions. On the other hand, not everything is necessarily traded in the balancing market since there is a risk of not being dispatched in this market. We find the value of waiting to be substantially larger than the cost of the additional risk, and so the gain from coordinated bidding is relatively small, suggesting that the incentive to enter the balancing market may be insufficient. In spite of this, coordinated bidding may become increasingly important in a future power system, given that the expected growth in fluctuating renewable production is expected to increase the value of balancing power.

Our model can be improved by the inclusion of risk measures in the objective or constraints in order to reflect a potential risk aversion of practitioners. Further improvements include more advanced price models, possibly integrating the bottom-up approaches often used in practice with our method based on statistics. Finally, our model can be modified to support bidding in other near real-time markets such as ancillary services markets. Following continuing discussions on the relevance of a so-called capacity market, an interesting extension of our model is the reservation of balancing power capacity on longer contracts.

Appendix A: Proof of Proposition 1

The lower bounds are obtained by deriving feasible solutions.

Since the optimal volumes bid and dispatched in the spot market, $\bar{\mathbf{x}}^{spot,+}, \bar{\mathbf{x}}^{spot,-}, \bar{\mathbf{y}}^{spot,+,n}$ and $\bar{\mathbf{y}}^{spot,-,n}$ for $n \in \mathcal{N}_2$ (the solution to the spot market bidding problem (8)), are feasible solutions to the coordination problem, we obtain the lower bound on total profit

$$z^i \geq \sum_{n \in \mathcal{N}_2} \pi^n \left(\sum_{t=1}^T \rho_t^n (\bar{y}_t^{spot,+,n} - \bar{y}_t^{spot,-,n}) + z^{reg,i,n}(\bar{\mathbf{y}}^{spot,+,n}, \bar{\mathbf{y}}^{spot,-,n}) \right).$$

If we further restrict the volumes bid and dispatched in the regulating market such that $\mathbf{x}^{reg,+,n-} = \mathbf{x}^{reg,-,n-} = \mathbf{y}^{reg,+,n} = \mathbf{y}_t^{reg,-,n} = 0$ for $n \in \mathcal{N}_3$, we have a lower bound on the profit in regulating bidding problem

$$\begin{aligned} & z^{reg,i,n'}(\bar{\mathbf{y}}^{spot,+,n'}, \bar{\mathbf{y}}^{spot,-,n'}) \\ & \geq \max \left\{ - \sum_{n \in \mathcal{N}_+(n') \cap \mathcal{N}_3} \pi^{n/n'} \left(\sum_{t=1}^T (\gamma_t^{i,-,n} z_t^{-,n} - \gamma_t^{i,+,n} z_t^{+,n}) + \mathcal{O}(q_1^n, \dots, q_T^n) \right) \mid \right. \\ & \quad \left. z_t^{-,n} - z_t^{+,n} = \bar{y}_t^{spot,+,n'} - \bar{y}_t^{spot,-,n'} - q_t^n, \quad t = 1, \dots, T, n \in \mathcal{N}_+(n') \cap \mathcal{N}_3 \right\} \\ & \quad := \underline{z}^{reg,i,n'}(\bar{\mathbf{y}}^{spot,+,n'}, \bar{\mathbf{y}}^{spot,-,n'}), \end{aligned}$$

for $n \in \mathcal{N}_2$.

Finally,

$$\begin{aligned} \sum_{n \in \mathcal{N}_2} \pi^n \left(\sum_{t=1}^T \rho_t^n (\bar{y}_t^{spot,+,n} - \bar{y}_t^{spot,-,n}) + z^{reg,i,n} (\bar{\mathbf{y}}^{spot,+,n}, \bar{\mathbf{y}}^{spot,-,n}) \right) \\ \geq \sum_{n \in \mathcal{N}_2} \pi^n \left(\sum_{t=1}^T \rho_t^n (\bar{y}_t^{spot,+,n} - \bar{y}_t^{spot,-,n}) + \underline{z}^{reg,i,n} (\bar{\mathbf{y}}^{spot,+,n}, \bar{\mathbf{y}}^{spot,-,n}) \right) = z^{spot,i}, \end{aligned}$$

by recognizing that the next to last term is the optimal value of the spot market bidding problem (8).

The above holds both when $\rho_t^n = \hat{\rho}_t^n, n \in \mathcal{N}_2$ and $\mu_t^n = \hat{\mu}_t^n, n \in \mathcal{N}_3$ (the market participant is a price-taker) and when allowing for price response (for any functional form of the price response).

□

Appendix B: Proof of Proposition 2

The derivation of upper bounds is based on a partial relaxation of non-anticipativity.

If we relax the bidding constraints (4)-(6) in the regulating market, we have an upper bound on total profit in the coordination problem

$$\begin{aligned} z^i \leq \max \left\{ \sum_{n \in \mathcal{N}_2} \pi^n \sum_{t=1}^T \rho_t^n (y_t^{spot,+,n} - y_t^{spot,-,n}) + \sum_{n \in \mathcal{N}_3} \pi^n \left(\sum_{t=1}^T (\mu_t^n (y_t^{reg,+,n} - y_t^{reg,-,n}) \right. \right. \\ \left. \left. - (\gamma_t^{i,-,n} z_t^{-,n} - \gamma_t^{i,+,n} z_t^{+,n})) - \mathcal{O}(q_1^n, \dots, q_T^n) \right) \mid (1) - (3), (7) \right\} := \bar{z}^i. \end{aligned}$$

Now, when $\rho_t^n = \hat{\rho}_t^n, n \in \mathcal{N}_2$ and $\mu_t^n = \hat{\mu}_t^n, n \in \mathcal{N}_3$, we have that $\gamma_t^{i,-,n} \geq \mu_t^n$ and $\gamma_t^{i,+,n} \leq \mu_t^n$ for $n \in \mathcal{N}_3$ and we further obtain

$$\begin{aligned} \bar{z}^i \leq \max \left\{ \sum_{n \in \mathcal{N}_2} \pi^n \sum_{t=1}^T \rho_t^n (y_t^{spot,+,n} - y_t^{spot,-,n}) - \sum_{n \in \mathcal{N}_3} \pi^n \left(\sum_{t=1}^T \mu_t^n ((y_t^{reg,-,n} + z_t^{-,n}) - (y_t^{reg,+,n} + z_t^{+,n})) \right. \right. \\ \left. \left. + \mathcal{O}(q_1^n, \dots, q_T^n) \right) \mid (1) - (3), (y_t^{reg,-,n} + z_t^{-,n}) - (y_t^{reg,+,n} + z_t^{+,n}) = \right. \\ \left. y_t^{spot,+,n} - y_t^{spot,-,n} - q_t^n \quad t = 1, \dots, T, n \in \mathcal{N}_3 \right\} = z^{spot,1}, \end{aligned}$$

by recognizing that the last problem is the spot market bidding problem (8) under a one-price balancing mechanism.

□

Endnotes

1. In comparison, the intra-day adjustment market Elbas, which is likewise operated by Nord Pool, had 100 members and a turnover of only 2.7 TWh.
2. It should be taken into account that part of the Danish demand for regulating power is usually covered by import from Norway and Sweden.

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